TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 4 2016

Calculator Free
Trigonometry

STUDENT'S NAME ______________________________

DATE: **TIME:** 50 minutes **MARKS:** 50

INSTRUCTIONS:

Standard Items: Pens, pencils, ruler, eraser.

Special Items: Notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (16 marks)

Prove the following identities

(a) $\dfrac{1-\sin x}{1+\sin x} = (\sec x - \tan x)^2$ [4]

$RHS = \left(\dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x}\right)^2$

$= \left(\dfrac{1-\sin x}{\cos x}\right)^2 = \dfrac{(1-\sin x)^2}{\cos^2 x}$

$= \dfrac{(1-\sin x)^2}{1-\sin^2 x}$

$= \dfrac{(1-\sin x)^{2}}{(1+\sin x)\cancel{(1-\sin x)}} = LHS$

OR

$RHS = \sec^2 x - 2\sec x\tan x + \tan^2 x$

$= \dfrac{1-2\sin x+\sin^2 x}{\cos^2 x}$

$= \dfrac{(1-\sin x)^2}{1-\sin^2 x}$

⋮ as before

$= LHS$

OR

$LHS = \dfrac{1-\sin x}{1+\sin x} \cdot \dfrac{1-\sin x}{1-\sin x}$

$= \dfrac{(1-\sin x)^2}{1-\sin^2 x}$

$= \dfrac{(1-\sin x)^2}{\cos^2 x} = \left(\dfrac{1-\sin x}{\cos x}\right)^2$

$= (\sec x - \tan x)^2$

$= RHS.$

(b) $$\frac{\sin 2A + \cos 2A + 1}{\sin A + \cos A} = 2\cos A$$ [4]

$$\text{LHS} = \frac{2\sin A\cos A + 2\cos^2 A}{\sin A + \cos A}$$

$$= \frac{2\cos A\,(\sin \cancel{A + \cos A})}{\sin \cancel{A + \cos A}} \qquad = \underline{\text{RHS}}$$

(c) $$\sec^2 A = \frac{\operatorname{cosec} A}{\operatorname{cosec} A - \sin A}$$ [4]

$$\text{RHS} = \frac{\frac{1}{\sin A}}{\frac{1}{\sin A} - \sin A} = \frac{\frac{1}{\sin A}}{\frac{1-\sin^2 A}{\sin A}} = \frac{1}{1-\sin^2 A}$$

$$= \frac{1}{\cos^2 A} = \sec^2 A = \underline{\text{LHS}}$$

(d) $$\frac{\sin 8A\ \cos A\ -\ \sin 6A\ \cos 3A}{\cos 2A\ \cos A\ -\ \sin 3A\ \sin 4A} = \tan 2A$$ [4]

$$\text{LHS} = \frac{\frac{1}{2}\left[\sin 9A + \sin 7A\right] - \frac{1}{2}\left[\sin 9A + \sin 3A\right]}{\frac{1}{2}\left[\cos A + \cos 3A\right] - \frac{1}{2}\left[\cos A - \cos 7A\right]}$$

$$= \frac{\sin 7A - \sin 3A}{\cos 3A + \cos 7A}$$

$$= \frac{\cancel{2}\cancel{\cos 5A}\sin 2A}{\cancel{2}\cancel{\cos 5A}\cos 2A} = \underline{\tan 2A}$$

2. (4 marks)

Using a suitable addition formula, calculate the exact value of $\cos 75°$

$$\cos 75 = \cos(30+45) = \cos 30 \cos 45 - \sin 30 \sin 45$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \underline{\frac{\sqrt{3}-1}{2\sqrt{2}}} \quad \left(= \frac{\sqrt{6}-\sqrt{2}}{4}\right)$$

3. (4 marks)

Show that $8\cos 80°\cos 40°\cos 20° = 1$

$$LHS = \frac{8}{2}\left[\cos 80(\cos 20 + \cos 60)\right] \quad \text{—①}$$

$$= 4\left[\cos 80 \cos 20 + \cos 80 \cos 60\right]$$

$$= \frac{4}{2}\left[\cos 60 + \cos 100 + \cos 20 + \cos 140\right]$$

$\cos 100 = -\cos 80$

$\cos 140 = -\cos 40$

$$= 2\left[\cos 60 + (\cos 20 - \cos 80 - \cos 40)\right]$$

$$= 2\left[\cos 60 + (\cancel{2}\sin 50 \cancel{\sin 30} - \cos 40)\right]$$

$$= 2\left[\cos 60 + (\cancel{\cos 40 - \cos 40})\right]$$

& $\sin 50 = \cos 40$

$$= 2 \cdot \frac{1}{2} = \underline{\underline{1}}$$

OR from ① $= 4\left[\cos 80\left(\cos 20 + \frac{1}{2}\right)\right]$

$$= 4\left[\cos 80 \cos 20 + \frac{1}{2}\cos 80\right]$$

$$= 4\left[\frac{1}{2}(\cos 60 + \cos 100) + \frac{1}{2}\cos 80\right]$$

$$= 2\left[\cos 60 - \cancel{\cos 80} + \cancel{\cos 80}\right]$$

$$= 1$$

4. (11 marks)

Solve the following equations for the given domains

(a) $2\sin\theta + \cos 2\theta = 1$, $0° \leq \theta \leq 180°$ [4]

$\Rightarrow 2\sin\theta + 1 - 2\sin^2\theta = 1$

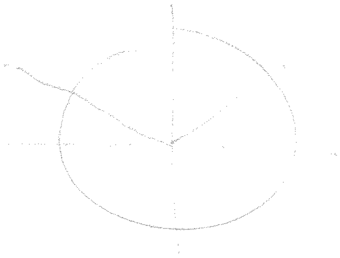
$\Rightarrow 2\sin^2\theta - 2\sin\theta = 0$

$\Rightarrow 2\sin\theta(\sin\theta - 1) = 0 \Rightarrow \sin\theta = 0$ or $\sin\theta = 1$

$\theta = 0°, 180°$ $\quad \theta = 90°$

(b) $\sin 2\theta = \sin\dfrac{\pi}{6}$, $0 < \theta < 2\pi$ [3]

$0 < 2\theta < 4\pi$



$2\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}$

$\Rightarrow \theta = \dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{13\pi}{12}, \dfrac{17\pi}{12}$

(c) $\tan\theta + 3\cot\theta = 5\sec\theta$, $0 < \theta < 2\pi$ [4]

$\dfrac{\sin\theta}{\cos\theta} + \dfrac{3\cos\theta}{\sin\theta} = \dfrac{5}{\cos\theta}$

$\times \sin\theta\cos\theta$

$\Rightarrow \sin^2\theta + 3\cos^2\theta = 5\sin\theta$

$\Rightarrow \sin^2\theta + 3 - 3\sin^2\theta = 5\sin\theta$

$\Rightarrow 2\sin^2\theta + 5\sin\theta - 3 = 0$

$\Rightarrow (2\sin\theta - 1)(\cancel{\sin\theta + 3}) = 0 \Rightarrow \sin\theta = \frac{1}{2}$

$\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

5. (7 marks)

(a) Show that the equation $\cos 3x - \sin 2x = 0$ can be written as $\cos x(4\sin^2 x + 2\sin x - 1) = 0$ [4]

$\Rightarrow 4\cos^3 x - 3\cos x - 2\sin x \cos x = 0$

$\Rightarrow \cos x(4\cos^2 x - 2\sin x - 3) = 0$

$\Rightarrow \cos x(4 - 4\sin^2 x - 2\sin x - 3) = 0$

$\Rightarrow \cos x(4\sin^2 x + 2\sin x - 1) = 0$

(c) Hence, by using the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, give the exact value for $\sin\dfrac{\pi}{10}$, if $\frac{\pi}{10}$ is a sol'n of the given eq'n. [3]

$\sin x = \dfrac{-2 \pm \sqrt{4 + 16}}{8}$

$= \dfrac{-1 \pm \sqrt{5}}{4}$

$\Rightarrow \sin\dfrac{\pi}{10} = \dfrac{-1 + \sqrt{5}}{4}$, since $\sin\frac{\pi}{10}$ is positive.

6. (8 marks)

If $A + B = \dfrac{\pi}{4}$ and $\tan A = \dfrac{n}{n+1}$, determine, in terms of n

(a) $\tan B$. (Hint: use the identity for $\tan(A+B)$) [4]

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1 \quad \text{since } \tan\frac{\pi}{4} = 1.$$

$$\Rightarrow \frac{n}{n+1} + \tan B = 1 - \frac{n}{n+1}\tan B$$

$$\Rightarrow \tan B\left[1 + \frac{n}{n+1}\right] = 1 - \frac{n}{n+1}$$

$$\Rightarrow \tan B\left[\frac{2n+1}{n+1}\right] = \frac{1}{n+1}$$

$$\Rightarrow \tan B = \frac{1}{2n+1}.$$

(b) $\tan(A - B)$ [4]

$$= \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\frac{n}{n+1} - \frac{1}{2n+1}}{1 + \left(\frac{n}{n+1}\right)\left(\frac{1}{2n+1}\right)}$$

$$= \frac{n(2n+1) - (n+1)}{(n+1)(2n+1) + n}$$

$$= \frac{2n^2 + n - n - 1}{2n^2 + 3n + 1 + n}$$

$$= \frac{2n^2 - 1}{2n^2 + 4n + 1}$$